$b \to ss\bar{d}$ in a Vector Quark Model

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Abstract

The rare decay $b \to ss\bar{d}$ is studied in a vector quark model by adding the contributions from exotic vector-like quarks. We find that the contribution from box diagrams amounts to 10^{-9} in the branching ratio, while the Z-mediated tree level contribution is negligible.

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1 Introduction

The flavor changing neutral current (FCNC) processes in B physics provide important windows to expose potential signals induced by new physics up to a scale around TeV. In the standard model (SM), the FCNC processes are induced at loop levels and are further suppressed by the Glashow-Maiani-Illiopoulous (GIM) mechanism. It is possible for the contribution from the new physics, whether at tree level or at loop level, to be competitive to its corresponding SM backgrounds. We could find out possible new physics through searching for the deviations from the SM predictions. It is not surprising that various FCNC processes are extensively studied in many of the extended models beyond the SM. Suppressed strongly in the SM as a second order weak process with strong GIM cancellations, the rare decay $b \to ss\bar{d}$[1] need to be considered as an important process which provides possible virtual signals of new physics. Many new physics models [2] with different flavor structures have shown the potential of enhancing significantly the branch ration of $b \to ss\bar{d}$. The corresponding exclusive decays (*e.g.* $B^- \to K^- K^- \pi^+$) have also been searched experimentally by different groups [3], which provides further constrains on the new physics models.

In this Letter, we investigate $b \to ss\bar{d}$ in a Vector Quark Model (VQM)[4]. With the inclusion of exotic heavy quarks with different quantum numbers under the SM gauge groups, it could be possible that the CKM matrix elements in the VQM are different and that the GIM cancellations in the first three generations are violated by these extra heavy quarks. Demanding all existed phenomenological constrains satisfied, we find that the branch ration of $b \to ss\bar{d}$ in a VQM could amount to 10^{-9}, several orders larger than its SM prediction which is below 10^{-12}[1].

2 Brief Review of Vector Quark Model

The VQMs are the SM extensions by adding into exotic quarks with non-standard $SU(2)_L \times SU(1)_Y$ assignments. The models could naturally emerge from some extensions of SM such as E_6 grand unified theory. Although these exotic quarks are heavy,

they do not necessarily decouple in the low energy phenomenology. At low energy they exhibit their effects through mixing with the ordinary quarks of the first three generations.

Here we focus on a simple model with one extra $Q = \frac{2}{3}$ up-type vector-like quark and one extra $Q = -\frac{1}{3}$ down-type vector-like quark, both of their left-hand and right-hand components translate as singlets under the SM gauge group. The ordinary and the vector-like quarks of the same electrical charges mix into the mass eigenstates which are denoted as

$$(u_{L,R})_\alpha = \begin{bmatrix} u_{L,R} \\ c_{L,R} \\ t_{L,R} \\ T_{L,R} \end{bmatrix}_\alpha, \quad (d_{L,R})_\alpha = \begin{bmatrix} d_{L,R} \\ s_{L,R} \\ b_{L,R} \\ B_{L,R} \end{bmatrix}_\alpha, \quad (\alpha = 1, 2, 3, 4). \tag{1}$$

They are related to the ordinary quarks in the weak eigenstates $u^0_{L,R}$ and $d^0_{L,R}$ by

$$(u^0_{L,R})_i = (U^u_{L,R})_{i\alpha}(u_{L,R})_\alpha, \quad (d^0_{L,R})_i = (U^d_{L,R})_{i\alpha}(d_{L,R})_\alpha, \quad (i = 1, 2, 3), \tag{2}$$

where $U^u_{L,R}$ and $U^d_{L,R}$ are all 3×4 matrices.

The charged current interactions in the mass eigenstates are:

$$\mathcal{L}_W = \frac{g}{2\sqrt{2}} \overline{u}_{L\alpha} \gamma^\mu V_{\alpha\beta} d_{L\beta} W^\dagger_\mu + h.c., \tag{3}$$

and

$$\mathcal{L}_{G^\pm} = \frac{g}{\sqrt{2}} \overline{u}_\alpha V_{\alpha\beta} \left[\frac{m_{u\alpha}}{M_W} P_L - \frac{m_{d\beta}}{M_W} P_R \right] d_\beta G^\dagger + h.c. \tag{4}$$

in the R_ξ gauge, where

$$V_{\alpha\beta} \equiv (U^{u\dagger}_L)_{\alpha i}(U^d_L)_{i\beta}, \quad (i = 1, 2, 3, \alpha, \beta = 1, 2, 3, 4) \tag{5}$$

is called the extended CKM matrix which is no longer unitary,

$$\begin{aligned} (V^\dagger V)_{\alpha\beta} &= \delta_{\alpha\beta} - (U^{d\dagger}_L)_{\alpha 4}(U^d_L)_{4\beta}, \\ (V V^\dagger)_{\alpha\beta} &= \delta_{\alpha\beta} - (U^u_L)_{\alpha 4}(U^{u\dagger}_L)_{4\beta}. \end{aligned} \tag{6}$$

The neutral current interactions are modified to

$$\mathcal{L}_{\mathcal{Z}} = \frac{g}{2C_W}(\overline{u}_{L\alpha}\gamma^\mu X^u_{\alpha\beta}u_{L\beta} - \overline{d}_{L\alpha}\gamma^\mu X^d_{\alpha\beta}d_{L\beta} - 2S_W^2\mathcal{J}^\mu_{EM})Z_\mu, \tag{7}$$

where

$$\begin{aligned}(X^u)_{\alpha\beta} &\equiv (U_L^{u\dagger})_{\alpha i}(U_L^u)_{i\beta} = (VV^\dagger)_{\alpha\beta}, \\ (X^d)_{\alpha\beta} &\equiv (U_L^{d\dagger})_{\alpha i}(U_L^d)_{i\beta} = (V^\dagger V)_{\alpha\beta}.\end{aligned} \tag{8}$$

The neutral interactions mediated by the goldstone boson G^0 are proportional to the quark masses. We will not display these small effects, as for the process discussed here is concerned. It is clear from (6 - 8) that Z-mediated FCNC interactions are induced at tree level in the VQM. In (7) the electromagnetic currents $\mathcal{J}_{EM}$ are the same as in the SM for the ordinary quarks. (See [4] for details.)

3 $b \to ss\bar{d}$

In the SM, the main contribution to the process $b \to ss\bar{d}$ is from the box diagrams with W and the up-quarks in the loops [1]. Due to the GIM mechanism, the amplitude is suppressed either by a small factor $V_{ts}^*V_{tb}V_{ts}^*V_{td}$, where V_{ij}'s stand for the (unitary) CKM matrix elements in the SM, or by a small power factor m_c^2/m_W^2. The resulting branching ratio is smaller than 10^{-12}.

In the VQM, $b \to ss\bar{d}$ can be induced by two mechanisms. One is the Z-mediated tree diagram. The other is the box diagrams with $W^\pm, G^\pm$ boson and the u, c, t, T quarks inside the loops. The Z-penguin diagrams are taken as higher order corrections to the tree diagram and their effects need not to be considered. We have

$$\begin{aligned}\Gamma_{VQM}(b \to ss\bar{d}) \;=\; & \frac{m_b^5}{48(2\pi)^3}\Bigg|\frac{G_F}{\sqrt{2}}X_{sb}X_{ds} + \frac{G_F^2}{2\pi^2}m_W^2\Bigg[X_{sb}X_{ds} \\ & + \sum_{\alpha=c,t,T}4X_{sb}\lambda^\alpha_{ds}B_0(x_\alpha) + \sum_{\alpha=c,t,T}4X_{ds}\lambda^\alpha_{sb}B_0(x_\alpha) \\ & + \sum_{\alpha,\beta=c,t,T}\lambda^\alpha_{sb}\lambda^\beta_{ds}S_0(x_\alpha, x_\beta)\Bigg]\Bigg|^2.\end{aligned} \tag{9}$$

We have denoted

$$\lambda^{\alpha}_{d_i d_j} = V^*_{\alpha d_i} V_{\alpha d_j}\,,\;\; x_\alpha = \frac{m_\alpha^2}{m_W^2}. \tag{10}$$

On the RHS of (9), the term outside the bracket represents the tree diagram contribution. In the bracket, the first term origins from the box diagram with two u-quarks in the loop; the second term is from the box diagram with u-quark connected to the s and b legs and c, t, T quarks connected to the d and s legs, while the third term comes from the box diagram with u-quark connected to the d and s legs and c, t, T quarks connected to s and b legs. The last term is from the box diagrams without u-quark in the loop. The Inami-Lim functions are [5]

$$\begin{aligned} F(x_\alpha, x_\beta) &= \frac{4-7x_\alpha x_\beta}{4(1-x_\alpha)(1-x_\beta)} + \frac{4-8x_\beta+x_\alpha x_\beta}{4(1-x_\alpha)^2(x_\alpha-x_\beta)} x_\alpha^2 \ln x_\alpha \\ &+ \frac{4-8x_\alpha+x_\alpha x_\beta}{4(1-x_\beta)^2(x_\beta-x_\alpha)} x_\beta^2 \ln x_\beta, \end{aligned} \tag{11}$$

$$S_0(x_\alpha) = F(x_\alpha, x_\alpha) - 2F(0, x_\alpha) + F(0,0), \tag{12}$$

$$S_0(x_\alpha, x_\beta) = F(x_\alpha, x_\beta) - F(0, x_\alpha) - F(0, x_\beta) + F(0,0), \tag{13}$$

$$4B_0(x_\alpha) = F(0, x_\alpha) - F(0,0). \tag{14}$$

4 Numerical Analysis

The VQM model are mostly constrained by $\Delta\mathcal{M}_K$, $\Delta\mathcal{M}_{B_d}$ and $\mathcal{B}r(B \to X_s\gamma)$. In the VQM, they can be expressed as:

$$\begin{aligned} \Delta\mathcal{M}_K^{VQM} &= \frac{G_F}{3\sqrt{2}} m_K (B_K F_K^2) \Big| \eta_Z^K X_{ds}^2 + \frac{G_F}{\sqrt{2}\pi^2} m_W^2 \Big[\eta_Z^K X_{ds}^2 \\ &+ \sum_{\alpha=c,t,T} 8X_{ds} \lambda^\alpha_{ds} \eta^K_{\alpha\alpha} B_0(x_\alpha) + \sum_{\alpha,\beta=c,t,T} \lambda^\alpha_{ds} \lambda^\beta_{ds} \eta^K_{\alpha\beta} S_0(x_\alpha, x_\beta) \Big] \Big|, \end{aligned} \tag{15}$$

$$\begin{aligned} \Delta\mathcal{M}_{B_d}^{VQM} &= \frac{G_F}{3\sqrt{2}} m_{B_d} (B_{B_d} F_{B_d}^2) \Big| \eta_Z^B X_{db}^2 + \frac{G_F}{\sqrt{2}\pi^2} m_W^2 \Big[\eta_Z^B X_{db}^2 \\ &+ \sum_{\alpha=t,T} 8X_{db} \lambda^\alpha_{db} \eta^B_{\alpha\alpha} B_0(x_\alpha) + \sum_{\alpha,\beta=t,T} \lambda^\alpha_{db} \lambda^\beta_{db} \eta^B_{\alpha\beta} S_0(x_\alpha, x_\beta) \Big] \Big|. \end{aligned} \tag{16}$$

As for the rare decay $B \to X_s\gamma$, it has been discussed in reference [6]. In the above equations, $\eta' s$ are the QCD factors. Here we take the values: $\eta_Z^K = 0.60$,$\eta_{cc}^K = 1.38$,$\eta_{tt}^K = 0.57$,$\eta_{ct}^K = 0.47$,$\eta_{TT}^K = 0.58$, $\eta_{cT}^K = 0.47$,$\eta_{tT}^K = 0.58$;$\eta_Z^B = 0.57$,$\eta_{tt}^B = \eta_{TT}^B = \eta_{tT}^B = 0.55$[7, 8]. Other parameters are $m_K = 498MeV$, $F_K = 160MeV$, $B_K = 0.86$, $m_{B_d} = 5.279GeV$,$F_{B_d}\sqrt{B_{B_d}} = 200MeV$[9].

It could be seen from (9) that Γ_{VQM} is parameterized by X_{sb}, X_{ds},$\lambda_{ds}^{c,t,T}$ and $\lambda_{sb}^{c,t,T}$. For simplicity, we take all the parameters as real in the numerical calculation. These parameters are not independent and can be related by the extended CKM matrix.

Numerical analysis is done in the following way. We take the upper sector of the extended CKM matrix as $V_{ud} = 0.9721, V_{us} = 0.215, V_{ub} = 2 \times 10^{-3}, V_{cd} = 0.209, V_{cs} = 0.966, V_{cb} = 3.8 \times 10^{-2}$. They are assumed to take their minimal values indicated by [9] so that the effects of the vector-like quarks can reach their maximum values. The other parameters are scanned in the regions of $200 < m_T < 800$, $0 < |V_{td}| < 0.09$, $0 < |V_{ts}| < 0.12$, $0.58 < |V_{tb}| < 0.99$,[9] $|X_{db}| < 0.0011$, $|X_{sb}| < 0.0011$ and $|X_{ds}| < 0.00001$[8]. Regarding Equation[5], we require $|0.9887 + V_{td}^2 + V_{Td}^2| < 1.0$, $|0.9794 + V_{ts}^2 + V_{Ts}^2| < 1.0$ and $|0.001448 + V_{tb}^2 + V_{Tb}^2| < 1.0$, and use these conditions to find out the ranges of V_{Td},V_{Ts} and V_{Tb}. The experimental constraints on $0 < \Delta\mathcal{M}_K^{VQM} < 2 \times 3.491 \times 10^{-15}$, $\left|\Delta\mathcal{M}_{B_d}^{VQM} - 3.2 \times 10^{-13}\right| < 0.092 \times 10^{-13}$, $|\mathcal{B}r(B \to X_s\gamma) - 3.15 \times 10^{-4}| < 0.54 \times 10^{-4}$ are demanded. The allowed parameter space is thus determined and branch ratio of $b \to ss\overline{d}$ is calculated.

Table 1: The maximum branch ration of $b \to ss\bar{d}$ vs m_T.

$m_T(GeV)$	200	400	600	800
$\Delta\mathcal{M}_K(GeV)$	6.948×10^{-15}	6.926×10^{-15}	6.758×10^{-15}	6.916×10^{-15}
$\Delta\mathcal{M}_{B_d}(GeV)$	3.234×10^{-13}	3.166×10^{-13}	3.257×10^{-13}	3.281×10^{-13}
$Br(B \to X_s\gamma)$	3.667×10^{-4}	3.532×10^{-4}	3.585×10^{-4}	3.683×10^{-4}
$Br(b \to ss\bar{d})$	1.841×10^{-10}	7.719×10^{-10}	1.308×10^{-9}	1.903×10^{-9}

We find that the contribution from the tree diagram amounts to only 10^{-15} in the

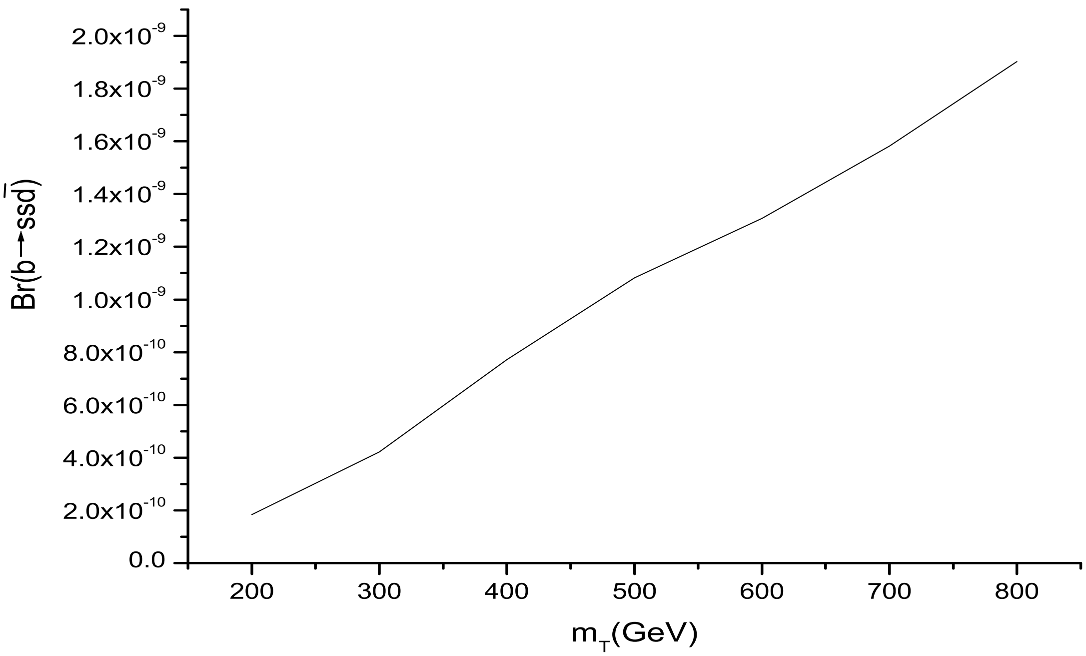

Figure 1: The maximum branch ratio of $b \to ss\bar{d}$. vs. m_T

branching ratio, which is even negligible compared to SM background. The effects of the box diagrams are the main contributions and the diagram with two T quarks dominates. In Table 1, we give the branching ratio of $b \to ss\overline{d}$ along with $\Delta\mathcal{M}_{B_s}$ by taking $X_{sb} = 0.0011$. We also plot the allowed branching ratio of $b \to ss\overline{d}$ as the function of m_T in Figure 1.

In conclusion, we have calculated the rare decay $b \to ss\bar{d}$ in the VQM and find its branching ratio could amount to 10^{-9}, about three orders of magnitude larger than its corresponding SM value. This work is supported in part by the National Natural Science Foundation of China (NSFC) under the grant No. 90103014 and No. 10205001, and by the Ministry of Education of China.

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